TRINITY COLLEGE

IN NOMINE DOMINI

Year 12 Methods Unit 3,4
Test 3 2021

Section 1 Calculator Free
DRV, Trigonometry, Growth & Decay, Rectilinear Motion

STUDENT'S NAME ______________________________

DATE: Thursday 13 May **TIME:** 20 minutes **MARKS:** 23

INSTRUCTIONS:
Standard Items: Pens, pencils, drawing templates, eraser

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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1. (7 marks)

The following table shows a discrete probability distribution for random variable X. The expected value of X is 2.

x	0	1	2	3	4
P(X = x)	a	a + b	0.2	0.3	0.1

(a) Determine the value of a and b [3]

$a + a + b + 0.2 + 0.3 + 0.1 = 1$

$2a + b = 0.4$

$a + b + 0.4 + 0.9 + 0.4 = 2$

$a + b = 0.3$

$2a + b = 0.4$
$-\quad a + b = 0.3$

$a = 0.1$
$b = 0.2$

(b) It is known that the standard deviation of X is 0.6 and for the random variable Y, Y = -2(X + 5). Determine

(i) E(Y) $= -2(2+5)$ [2]

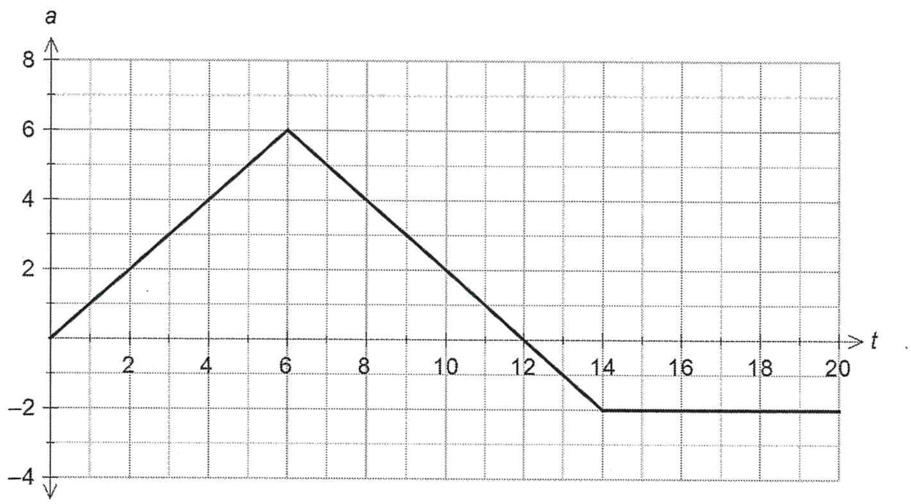
$= -14$

(ii) SD(Y) $= |-2|\, 0.6$ [2]

$= 1.2$

2. (8 marks)

A particle, initially at rest at the origin, moves subject to an acceleration, a ms^{-2}, as shown in the graph below for $0 \le t \le 20$ seconds.



(a) Determine the velocity of the particle when

(i) t = 6   18 m/s [1]

(ii) t = 20 $\frac{12\times6}{2} - \frac{2\times2}{2} - 2\times6$ [2]

$= 22$ m/s

(b) At what time is the velocity of the particle a maximum, and what is the maximum velocity? [2]

$t = 12$ $v = 36$

(c) Determine the distance of the particle from the origin after 3 seconds. [3]

$a = t$

$v = \int t\,dt$

$v = \frac{t^2}{2} + c$

$\begin{pmatrix} v=0 \\ t=0 \end{pmatrix}$

$\therefore c = 0$

$v = \frac{t^2}{2}$

$x = \int \frac{t^2}{2}\,dt$

$x = \frac{t^3}{6} + c$

$x=0$
$t=0$

$\therefore c = 0$

$x = \frac{t^3}{6}$

$t=3$ $x = \frac{27}{6}$

3. (5 marks)

(a) Differentiate $7x\sin(3x)$ with respect to x. [2]

$$\frac{d}{dx} 7x \sin 3x = 7\sin 3x + 7x\cos 3x \times 3$$

$$= 7\sin 3x + 21x\cos 3x$$

(b) Hence determine $\int x\cos(3x)dx$ [3]

$$\frac{d}{dx} 7x\sin 3x = 7\sin 3x + 21x\cos 3x$$

$$\int \frac{d}{dx} 7x\sin 3x \, dx = \int 7\sin 3x \, dx + \int 21x\cos 3x \, dx$$

$$7x\sin 3x = -\frac{7\cos 3x}{3} + 21\int x\cos 3x \, dx$$

$$c + \frac{7x\sin 3x}{21} + \frac{7\cos 3x}{63} = \int x\cos 3x \, dx$$

4. (3 marks)

For a $5 monthly fee, a TV repair company guarantees customers a complete service. The company estimates the probability that a customer will require one service call in a month as 0.05, the probability of two service calls as 0.01 and the probability of three or more calls as 0.00. Each call costs the repair company $40. What is the TV repair company's expected monthly gain from each customer?

CALLS	0	1	2	≥ 3
g (GAIN)	5	-35	-75	0
$P(G = g)$	0.94	0.05	0.01	0

$$E(G) = 5 \times 0.94 + (-35)0.05 + (-75)0.01$$

$$= 2.2$$

$2.20 GAIN

TRINITY COLLEGE

Year 12 Methods Unit 3,4
Test 3 2021

Section 1 Calculator Assumed
DRV, Trigonometry, Growth & Decay, Rectilinear Motion

STUDENT'S NAME ______________________________

DATE: Wednesday 31st March **TIME: 30** minutes **MARKS**: 32

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser

Special Items: Three calculators, notes on one side of a single A4 page (these notes to be handed in with this assessment)

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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5. (8 marks)

From the analysis of median house price (M) in a city on July 1 each year from 2001 to 2019, it was determined that $\frac{dM}{dt} = 0.0746M$, where t is the time in years since July 1 2001.

(a) According to this model, how long does it take for house prices to double? [3]

$$M = M_0 e^{0.0746t}$$

$$2\cancel{M_0} = \cancel{M_0} e^{0.0746t}$$

$$2 = e^{0.0746t}$$

$$t = 9.3 \text{ YRS}$$

From the data collected, it was observed that the median house price on July 1 2009 was $430 000.

(b) Determine the instantaneous rate of change of the median house price at this time. [1]

$$\frac{dM}{dt} = kM$$

$$= 0.0746 \times 430000$$

$$= \$32078$$

(c) What was the median house price July 1 2018? [2]

$M = 430\,000$

$t = 8$

$$430\,000 = M_0 e^{0.0746 \times 8}$$

$$236745 = M_0$$

$(t = 17)$ $$M = 236745 \times e^{0.0746 \times 17}$$

$$= \$841497$$

(d) When did the median house price reach $500 000? [2]

$$500\,000 = 236745 e^{0.0746t}$$

$$t = 10 \text{ YRS}$$

6. (12 marks)

Analysis of the number of dogs registered by each household within a suburb resulted in the following percentages.

Number of dogs registered	0	1	2	3 or more
Percentage of households	21	44	27	8

(a) A council worker selects households at random to visit. What is the probability that the first five households visited all have at least one dog registered? [2]

$b(5, 5, 0.79) = 0.3077$

(b) A random sample of 40 households within the suburb is selected. Use the binomial distribution to determine the probability the sample contains:

(i) exactly 6 households with no registered dogs [2]

$b(6, 40, 0.21) = 0.1088$

(ii) no more than 15 households with at least two registered dogs [2]

$b(0 \le d \le 15, 40, 0.35) = 0.6946$

(c) A random sample of 25 households within the suburb is to be selected. If the random variable D is the number of households in the sample that have exactly one dog registered, determine E(D) and Var(D). [3]

$E(D) = np$
$= 25 \times 0.44$
$= 11$

$VAR(D) = np(1-p)$
$= 11 \times 0.56$
$= 6.16$

(d) In a random sample of 50 households, where the random variable Z is the number of households with no registered dogs, determine $P(Z \ge 2 | Z \le 4)$. [3]

$$\frac{P(2 \le Z \le 4)}{P(Z \le 4)} = \frac{0.0122}{0.0123}$$
$$= 0.9919$$

7. (12 marks)

Sam is one of ten members of a social club. Each week one member is selected at random to win a prize.

(a) Define the random variable. [1]

$X \sim b(n, 0.1)$

(a) What is the probability that Sam has his third win in week 8. [2]

$b(2, 7, 0.1) \times 0.1$

$= 0.1240 \times 0.1$

$= 0.0124$

(b) In the first 20 weeks, does Sam have a greater chance of winning exactly 2 prizes or exactly 3 prizes? [3]

$b(2, 20, 0.1)$

$= 0.2852$

$b(3, 20, 0.1)$

$= 0.1901$

$\therefore$ 2 PRIZES GREATER

(c) For how many weeks does Sam have to participate in the prize draw so that he has a greater chance of winning exactly 3 prizes than of winning 2 prizes? [3]

Note: $\binom{n}{r} = \frac{n!}{r!(n-r)!}$

$b(2, n, 0.1) < b(3, n, 0.1)$

$\binom{n}{2} 0.1^2 \, 0.9^{n-2} < \binom{n}{3} 0.1^3 \, 0.9^{n-3}$

$\frac{n(n-1)}{2!} 0.1^2 \, 0.9^{n-2} < \frac{n(n-1)(n-2)}{3!} 0.1^3 \, 0.9^{n-3}$

SOLVE TO GET $n = 30$

(d) For how many weeks must Sam participate so that the probability he wins at least once is at least 0.9? [3]

$b(\geq 1, n, 0.1) \geq 0.9$

ie $b(0, n, 0.1) < 0.1$

$\binom{n}{0} 0.1^0 \, 0.9^n < 0.1$

$n = 21.85$

ie $n = 22$